STOCHASTIC OPTIMIZATION LIMITED

EFFECTIVE MANAGEMENT OF UNCERTAINTY

Supplement to

Report on Locational Price Risk Management

by

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for

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[1] This report was commissioned by the Electricity Authority as a supplement to the Stochastic Optimization Report on Locational Risk Management, July 22, 2011. Version 15 of this supplement, dated September 12, 2011, corrects a numerical error in the example in section 4 of version 14, dated August 31, 2011. (We are grateful to Roger Miller of the Electricity Authority for making this known to us.) Stochastic Optimization Limited is a private company, and the results and conclusions of this report are based on models developed by Stochastic Optimization specifically for the purpose of this report. The results and conclusions of this report are not associated with the Electric Power Optimization Centre or with the University of Auckland.

1. Introduction

Stochastic Optimization submitted a Report on Locational Price Risk Management (which we call the SOL LPRM Report) to the Electricity Authority on July 22, 2011. This report commented on aspects of Schedule 14.6, which has subsequently been revised. This supplementary report examines the latest revision of 14.6 (as current on August 10, 2011), and discusses the calculation of the loss and constraint excess that is to be paid into the Financial Transmission Right (FTR) account under clause 14.73(2A) of the Code.

This supplementary paper will comment on:

(a) the correctness of mathematical formulae used in the schedule 14.6; and

(b) whether the judgments used in schedule 14.6 are reasonable, within the context of the assumptions made and background to the purpose of schedule 14.6 already provided to the consultant.

The mathematical formulae in schedule 14.6 are intended to determine the amount to be paid into the FTR account for each trading period. These amounts are intended to be sufficient to cover the congestion payments to any point-to-point FTR contracts which are extant at the time of dispatch over some set of hubs[2]. The correctness of these formulae must be measured in terms of this intent.

Our interpretation of the correctness of 14.6 is therefore the following: if the *on-the-day* (otd) transmission grid is the same as the transmission grid determining FTR quantities then the formulae can then be said to be "correct" if they provide enough revenue to cover the congestion payments from any FTRs that have been allocated that are in aggregate simultaneously feasible for the grid constraints.

This report will look at the schedule in detail in section 4. Section 2 and section 3 discuss the general approach and its correctness. We do not propose remedies to any problems that might be identified here, but simply point out features of Schedule 14.6 that might require some attention when specified in more detail.

The amount that accrues in rentals can be divided up into rentals from the AC network and rentals from the HVDC line. We will discuss AC rentals in Section 2 and HVDC rentals in Section 3.

2. AC rentals

As outlined in Appendix 2 to the SOL LPRM report, AC rentals accrue from a vector f of flows in the AC network on the day of dispatch. The total rental pool from AC lines in a trading period of duration 0.5 hours is then

$$0.5\sum_i \pi_i \, g_i(f)$$

[2] Although these are not formally defined in Schedule 14.6, it is helpful to think of these as Benmore and Otahuhu grid exit points.

where $g_i(f)$ is the net flow into node i from the AC network, and π_i is the final price at node i.

Revenue adequacy using the simultaneous-feasibility test relies on being able to separate the constraints of the economic dispatch problem into two sets, those containing only flow variables, and those containing only other variables. In the simultaneous feasibility test, the only constraints that are allowed to have both types of variables are the flow conservation constraints at each node.

Introducing mixtures of the variables in constraints violates this separation. This means that the mixed constraints in SPD will require special attention in a rental allocation. The HVDC also requires special attention as it may set the risk in reserve constraints.

The AC rentals from the constraints containing only flow variables can be decomposed in terms of shadow prices and right-hand sides of constraints[3]. Thus for line k one might model the flow f_k as

$$f_k = r_k + s_k - r'_k - s'_k$$

where r_k and s_k correspond to flow intervals with different marginal losses in a piecewise linear representation, and r'_k and s'_k are flows in the opposite direction. Here we have

$$0 \le r_k \le u_k,\ \ 0 \le s_k \le v_k,\ \ 0 \le r'_k \le u'_k,\ \ 0 \le s'_k \le v'_k$$

where u_k and v_k and u'_k and v'_k correspond to different loss tranches. Denoting the shadow prices of these constraints by μ_k and λ_k and μ'_k and λ'_k we have

$$\Sigma_i \pi_i\, g_i(f) = \Sigma_k u_k\, \mu_k + \Sigma_k v_k\, \lambda_k + \Sigma_k u'_k\, \mu'_k + \Sigma_k\, v'_k\, \lambda'_k$$

Similar expressions can be derived for all constraints involving only transmission flows, such as security constraints.

Given constraint shadow prices, the total rental can be accumulated by examining each constraint. A key ingredient of Schedule 14.6 is to allocate a portion of rent from each constraint to the FTR account. The sum of all these portions is intended to be enough to fund congestion payments to any set of FTRs that have been assigned within a convex set defined by extreme FTR flow patterns.

To determine the portions that accrue to each constraint, a parameter called *AssignedCapacity* is introduced to determine the rentals that should be assigned to a given FTR flow pattern in the network. The parameter AssignedCapacity takes a portion of the rentals from each constraint so that they correspond to the fraction of the flows in that constraint that would arise from the FTR flow pattern. AssignedCapacity is defined in terms of shift factors, or Power Transfer Distribution Factors (PTDFs) for a lossless (balanced) flow. The lossless flow is defined to be an

[3] This is derived for a general dispatch problem in Section 4 of Appendix 2 to the SOL LPRM report.

approximation of a flow with losses (i.e. an unbalanced flow). The lossless approximation is required to exceed the flow with losses in every branch.

The algorithm by which AssignedCapacity is computed is as follows.

1. The FTR manager determines some unbalanced FTR patterns that are simultaneously feasible with losses;
2. The FTR manager approximates each of these by a balanced FTR pattern that gives branch flows that exceed the branch flows from the unbalanced FTR pattern. (It is not stated how this approximation is to be done).
3. These balanced FTR patterns are used to determine AssignedCapacity for each constraint, by estimating what portion of the constraint is used by flows from a balanced FTR in a lossless network.

The approximation procedure in step 2 has not been defined in Schedule 14.6. We discuss how this might be done in more detail in section 4.

The algorithm for determining AssignedCapacity depends on the calculation of PTDFs. This must be done ex-ante, in other words, prior to dispatch. The computation of exact PTDFs prior to dispatch is only possible for flows without losses. This is why the algorithm uses a lossless flow pattern that approximates the branch flows from the unbalanced FTR pattern.

We have not proved that the approach in Schedule 14.6 will collect enough rentals to fund any FTR pattern that is in the convex hull of the extreme balanced FTR patterns that are constructed using the approximation procedure. This relies on at least the provision of some mechanism to account for the reserve implications of the HVDC setting the risk, and some mechanism to deal with the cost of losses. Both of these issues are already under consideration by the Electricity Authority.

3. HVDC rentals

The HVDC rental formula in Schedule 14.6 is:

$$\max\left(\begin{array}{l} 0, \sum_{n(NI)} price_n \times \left(\sum_{l \in R_{HVDC}(n)} \left(HVDCLinkFlow_l - HVDCLinkLosses_l \right) - \sum_{l \in S_{HVDC}(n)} HVDCLinkFlow_l \right) \\ + \sum_{n(SI)} price_n \times \left(\sum_{l \in R_{HVDC}(n)} \left(HVDCLinkFlow_l - HVDCLinkLosses_l \right) - \sum_{l \in S_{HVDC}(n)} HVDCLinkFlow_l \right) \end{array}\right) \div 2$$

This is consistent with how the HVDC is modelled in SPD. The term in brackets is the same as

$$R_t(\text{HVDC}) = \sum_{n(SI)} \pi_n g_n(f) + \sum_{n(NI)} \pi_n g_n(f)$$

as defined in Appendix 2 to the SOL LPRM report. This is the constraint rental accruing to the HVDC.

As discussed in Appendix 2 to the SOL LPRM report, some care must be taken in the case where the HVDC is setting the reserve risk in the dispatch. The congestion payment that must be made to an unbalanced simultaneously feasible FTR might exceed R_t(HVDC). An example that demonstrates this is given in Appendix 3.

In the sense defined above the formula is not "correct" as it may leave a shortfall in revenue. (It is of course correct in that it records the correct value of R_t(HVDC).)

The issue here (like accounting for losses) has been considered by the Electricity Authority in their consultations on Financial Transmission rights. Possible solutions involve contracting for reserve support to provide any revenue shortfall when FTRs are allocated, or restricting the volume of FTRs allocated to be less than the risk-setting flow on the HVDC.

The formula above does not account for a situation where the HVDC sets the reserve risk, while its rentals are being used to fund the reserve for this. The implication is that if this reserve cost is to be funded from HVDC rentals, then the above formula will overstate the rentals to be transferred into the FTR account, unless the link flow is severely restricted. (In the example in Appendix 3 we would need to set this to be zero to transfer $0 to the FTR account.)

4. Detailed comments on Schedule 14.6

Clause 2 Interpretation

simultaneously feasible alludes to constraints in clause 5(8). This subclause does not exist.

unbalanced FTR injection pattern

The revenue adequacy theorem requires a convex dispatch problem. This means that the feasible region allows free disposal of energy at the nodes. In other words, flow balance constraints at grid exit points must be inequality constraints rather than equations. The reader is referred to Appendix 2 to the SOL LPRM report for a proof of revenue adequacy under this assumption. Allowing free disposal of power means that all nodal prices are non-negative at optimality. If all flow balance constraints are equations then it is easy to construct instances with negative nodal prices for which the revenue adequacy theorem fails[4].

In practice, of course, SPD is solved assuming no free disposal, and so negative prices can occur. This can put revenue adequacy at risk in these cases for some FTR patterns, even though they are simultaneously feasible. It could be argued that these circumstances are sufficiently rare to be ignored. However this issue is important when considering what FTRs might be allocated in an auction, and therefore what funding should be assigned in Schedule 14.6 to support these.

[4] See e.g. Philpott, A.B. and Pritchard, G., Financial transmission rights in convex pool markets, Operations Research Letters, 32 (2004) 109 – 113.

In particular, the issue of whether to admit free disposal affects the definition of an extreme unbalanced FTR injection pattern. Extreme points are defined for convex sets. The intention in Schedule 14.6 is that any FTR injection pattern that is a convex combination of extreme FTR injection patterns is a feasible FTR injection pattern. This will only be the case if the convex set of feasible FTR patterns admits free disposal of energy at any node. The 50-50 convex combination of unbalanced FTRs (-3,1) and (1,-3) is (-1,-1), which can only be a feasible FTR pattern if some energy is spilt somewhere.

Thus, if the intention of Schedule 14.6 is to construct a convex feasible set of unbalanced FTR patterns, then the FTR patterns must admit free disposal. This then affects how the extreme points should be defined.

To see the difference between flow patterns with free disposal and without, consider an unbalanced FTR between nodes 1 and 3 in the network in Figure 1 below.

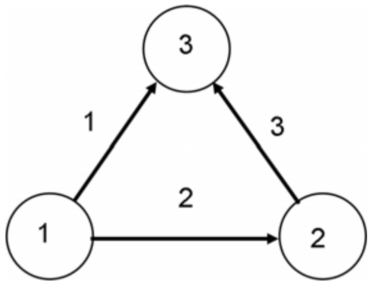

Figure 1: Network example

Suppose all branches have reactance 1 and suppose the capacity of lines 1 and 2 is 4, and that of line 3 is 1 unit. Let the loss factor on line 1 be 50%. Then an unbalanced FTR pattern from 1 to 3 is (-3, 2), giving a flow f = (2,1,1). This might be thought of as an extreme point for the problem with no free disposal. If so, by symmetry the unbalanced FTR pattern from 3 to 1 of (-3, 2) is also an extreme point with flows f = (-2,-1,-1). The average of these two FTRs is (-0.5,-0.5) from 1 to 3. This is not a feasible FTR with no free disposal, showing that the set of FTRs not admitting free disposal is not convex.

If we allow free disposal of energy at any node, then we can get an FTR from 1 to 3 of (-7, 3) with flows f = (4, 3, 1). This sheds two units of flow at node 2, and is an extreme point of the (convex) set of FTR patterns that allow free disposal.

5. Review of each clause in Schedule 14.6

Clause 4

This specifies a normal grid configuration to be supplied by the system operator to the FTR clearing manager

Clause 5

(1) & (2) FTR manager determines unbalanced extreme FTR injection patterns

It is not specified exactly how this is to be done, and it depends on the discussion of unbalanced FTR pattern and free disposal as above.

FTR manager selects a set of balanced FTRs that approximates the set of unbalanced FTRs.

For each unbalanced FTR injection pattern, the FTR manager selects a balanced FTR injection pattern that approximates it. This approximation is not specified, but it should produce flow on every line that is no less than the flow from the unbalanced FTR injection pattern. Ideally it should take the same value at the FTR injection point as the unbalanced FTR injection pattern.

The best balanced FTR injection pattern could be determined by an optimization problem. Suppose the unbalanced FTR injection pattern generates a flow in each line k equal to u_k. Suppose we adopt the convention that the node-arc incidence matrix A of the network is chosen so that $u_k \geq 0$. We seek a vector of flows b with the property that $b_k \geq u_k$, and meeting the flow conservation constraints of the network (ignoring losses and capacities).

Let L be a loop matrix for the network (which has rows corresponding to oriented loops in the network, and a columns corresponding to each branch. The entries $(L)_{ij}$ are +1, -1 or 0 depending on whether the flow variable f_j for the branch j is oriented in the same direction as loop i, or opposite, or is not part of the loop. Let C be a diagonal matrix of branch reactances. This leads to a set of equations

$$LCf=0$$

that represent Kirchhoff's voltage law in the DC load-flow approximation. The constraints LCf=0 arise because flow around opposite sides of any loop must respect the line impedances.

Suppose that r is a vector with components equal to zero at all nodes except at the hubs i and j where it equals the balanced FTR amount. In other words r is negative at the FTR injection point, node i, and positive with the same magnitude at the FTR offtake point, node j. Given the unbalanced FTR injection pattern u, suppose that we further require $-r_i$ to be the same injection at node i as the unbalanced FTR. Mathematically,

$$r_i = -e_i^T Au,$$

where e_i is a unit vector with 1 in the ith row. So this gives $r_j = e_i^T Au$, and $r_m=0$ for nodes $m \neq i,j$.

Then, given this r, we could try to solve

$$\begin{aligned} P &: \min \sum_k b_k \\ s.t \quad & b_k \geq u_k \\ & Ab = -r \\ & LCb = 0 \end{aligned}$$

to find a best balanced approximation. Here Ab=-r says that the flows add to zero at all nodes except i and j where they equal the (balanced) FTR amounts. Unfortunately, if we require

$$r_i = e_i^T Au$$

then P does not always have a feasible solution, even if r_i gives a feasible flow with losses. (See Appendix 1 for an example.)

It is possible to obtain a feasible solution by relaxing the requirement that r_i is the same value as $e_i^T Au$. In other words if the directions of branches in the network are chosen so that b_k and u_k are taken as positive then we can scale any solution (r,b) to

$$\begin{aligned} Ab &= -r \\ LCb &= 0 \end{aligned}$$

until $b_k \geq u_k$ in every component. This might require a balanced FTR r that is substantially larger than the unbalanced FTR we started with.

An alternative is to admit unbalanced FTR injection patterns between more nodes than the hubs defined in the Code. This is explored in Appendix 2[5].

Clause 6

The shift factor calculation for lossless flows is based on the standard DC - Load flow approximation, and is correct.

Clause 7

[5] We are not recommending a change to the Code here. We are merely alerting the reader to the fact that a good approximation of an unbalanced flow pattern might not always be available under the assumptions made by the Code.

For each trading period the FTR manager decides a branch participation loading for each line

Branch participation loading

The branch participation loading is computed over a set of P injection patterns that are chosen to represent the extreme FTR flow patterns. This set patterns is not defined precisely but is left up to the FTR manager.

If a single FTR pattern is chosen (i.e. P=1) that gives a flow that is opposite to the scheduled flow, then even if the scheduled flow is in the forward flow direction, the branch participation loading will yield a negative result. The parameter AssignedCapacity is the minimum of this and the actual capacity, so the result will be negative. This makes it possible for a negative rental to be transferred to the FTR account which is probably not the intent of the construction. To avoid this problem, the FTR provider must select from a rich enough set of extreme FTR patterns to cover all possible scheduled flow directions. Otherwise, there is a risk of underestimating the rentals.

With enough extreme patterns, the formulae could overestimate on-the-day participation, but provides a bound which will possibly collect more rental revenue from the dispatch than is needed. This is preferable to collecting less rental than is required.

Constraint participation loading (branch constraints)

If P=1, then the same problem as above, where a negative rental is transferred to the account, might occur.

As above, the formulae could also overestimate on-the-day participation, but provides a bound which will possibly collect more rental revenue from the dispatch than is needed.

Constraint participation loading (mixed constraints)

If P=1, then the same problem as above, where a negative rental is transferred to the account, might occur.

As above, the formulae could overestimate on-the-day participation, but provides a bound which will possibly collect more rental revenue from the dispatch than is needed.

Clause 8

The FTR manager determines the parameter *AssignedCapacity*, which is a portion of capacity for each constraint RHS. The portion is the minimum of the constraint RHS and the branch participation loading. These are computed separately for AC lines, AC line loss curve blocks, branch constraints and mixed constraints.

Clause 9

This clause defines the amounts to be paid into the FTR account. These are derived from the "Assigned Capacities" from Clause 8, and the shadow prices of the constraints. The Assigned Capacities bound the largest possible use of the line by any FTR flow that is the convex combination of extreme FTR patterns. The intention is to ensure that enough revenue is collected to cover the congestion payments for all FTRs that might be allocated. There is of course a possibility that this construction will collect too much rental. We have not studied how material this might be.

(2) The HVDC line has been discussed in section 3 above.

(3) and (4) Subject to the caveat on ensuring a nonnegative branch participation factor as discussed above, AssignedCapacity times the shadow price times 0.5 gives the correct rental amount per trading period,.

(5) Clause 9(5) defines the portion of the loss rental that is accrued by each line that must be transferred into the FTR account.

The formula for the marginal loss factor ($ACLinelossFactor_{k,marg}$) is taken to be the maximum loss gradient at the flow when this is not uniquely defined.

The formula is consistent with the loss rental formula derived in Appendix 2 (page 21) to the SOL LPRM report.

Observe that in the case that the line is at its thermal limit the rental must be increased by 0.5 times the shadow price of this constraint times its Assigned Capacity (as defined correctly in Clause 9(4)).

Appendix 1: P can be infeasible

In this appendix we show that approximating an unbalanced FTR pattern (injecting r_i at node i) by a balanced FTR pattern with the same injection at node i might not be possible. Consider the problem

$$
\begin{aligned}
P: &\min \sum_k b_k \\
s.t \quad & b_k \ge u_k \\
& Ab = -r \\
& LCb = 0
\end{aligned}
$$

where A is the node-arc incidence matrix for the network in Figure 2.

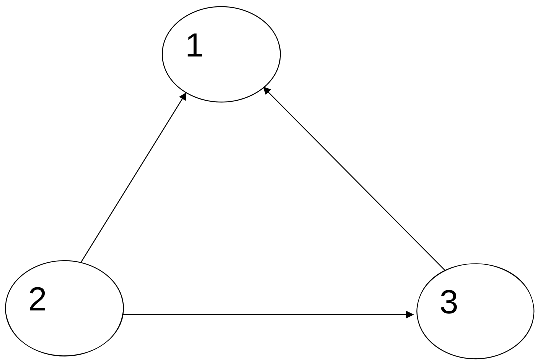

Figure 2

Suppose all reactances are 1 and there is a 50% loss factor on line (2,3). Then an unbalanced FTR between 2 and 1 is (-10,8). This gives a u_{21}=6, u_{23}=4, and u_{31}=2. There is no feasible flow b to support a balanced FTR between 1 and 2 of (-10,10) that is no less than u_k in every branch k, even if we ignore thermal limits. This is because, if $u_{23} \ge 4$ then $u_{31} \ge 4$ follows from flow conservation, and so to satisfy Kirchhoff's Laws, $u_{21} \ge 8$. This gives an FTR of at least (-12, 12).

Appendix 2: Approximating an unbalanced FTR

In this appendix we show that there is always a best balanced approximation to an unbalanced FTR injection pattern if we allow more nodes to be hubs. The construction works by setting the sent balanced flow in every branch to be equal to the sent unbalanced flow. The imbalance created (from losses) at each node is then imposed as a negative injection at this node. The result is a balanced FTR injection pattern, but with many hubs.

Formally this construction is as follows. Suppose u is a vector of flows from an unbalanced FTR, and the lossless node-arc incidence matrix A of the network is chosen so that $u_k \geq 0$. Here u_k denotes the flow leaving a node at the start if branch k. Let $T_k \geq 0$ denote the thermal loss on each line.

The vector u can be seen to be the solution to a linear system of equations

$$Gu=-r$$

where G is a generalized node-arc incidence matrix. Thus each column of G corresponds to a branch flow, and has a 1 in the row corresponding the upstream node and a $(-1+T_k/u_k)$ corresponding to the downstream node. The vector r is an unbalanced FTR with $r_i<0$, and $r_j>0$, and $r_m=0$, $m \neq i,j$. The magnitude of r_j is the same as that of r_i minus the losses.

Now define a balanced FTR by $s_i=r_i$, $s_j=r_j+e_j^T(G-A)u$, and $s_m= e_m^T(G-A)u$, $m \neq i,j$. This is balanced because

$$Au=Gu - (G-A)u$$

so

$$e_i^TAu = e_i^TGu - e_i^T(G-A)u = e_i^TGu = -r_i = -s_i$$

$$e_j^TAu = e_j^TGu - e_j^T(G-A)u = -r_j - e_j^T(G-A)u = -s_j$$

$$e_m^TAu = e_m^TGu - e_m^T(G-A)u = -s_m$$

which shows that the components of the FTR sum to 0, since the rows of Au sum to zero.

Also b=u is a (balanced) optimal solution to P with FTR s, and gives the best balanced approximation to the unbalanced FTR. This amounts to adding an offtake of T_k at the downstream endpoint of every line k, and setting the lossless flow in each line to be u_k .

Appendix 3: HVDC Revenue Inadequacy with Reserve

This spreadsheet displayed in Figure 3 shows a solution in which a simultaneously feasible balanced FTR cannot be funded from constraint rentals. There are no losses in this model, but the single branch sets the risk to be met by reserve in the South.

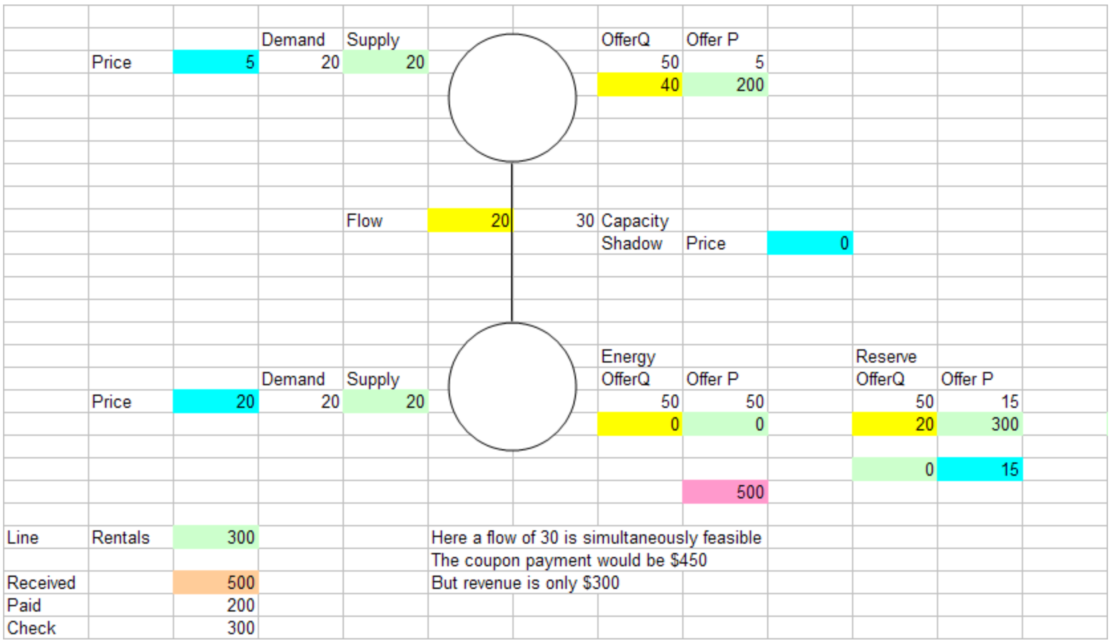

Figure 3: Revenue inadequacy with HVDC setting reserve

There is a flow from North to South of 20, which sets the risk as there is no dispatched generation in the South. The reserve price is $15/MWh which is the price difference between the nodes. The loads pay $400+$100 for their energy. The North generator is paid $200 for its energy. The total transmission rental is $300. Observe that this is the same as $R_t(HVDC)=(\pi_j-\pi_i)f_{ij}$= ($20 - $5)*20 = $300.

A flow of 30 from North to South is simultaneously feasible. Its congestion payment is ($20 - $5)*30= $450 which cannot be funded from R_t (HVDC).

Observe that this inadequacy is not because of the $300 that the system must pay for the reserve provided. By Lemma 5 in Appendix 2 of the SOL LPRM report we see that R_t(HVDC) will cover this cost, but this is not the issue here. The issue is that an FTR of (-30,30) might have been sold on the (N-S) branch, and this cannot be funded from R_t (HVDC), even if we have some means of paying the $300 to the reserve provider from elsewhere.

One might claim that the $300 in transmission rental must be paid to the reserve provider, so that there will be no rentals available to support an FTR. On the other hand, Clause 9 of Schedule 14.6 allocates

$$\max\left(\begin{array}{l}0, \sum_{n(NI)} price_n \times \left(\sum_{l \in R_{HVDC}(n)} \left(HVDCLinkFlow_l - HVDCLinkLosses_l \right) - \sum_{l \in S_{HVDC}(n)} HVDCLinkFlow_l \right) \\ + \sum_{n(SI)} price_n \times \left(\sum_{l \in R_{HVDC}(n)} \left(HVDCLinkFlow_l - HVDCLinkLosses_l \right) - \sum_{l \in S_{HVDC}(n)} HVDCLinkFlow_l \right)\end{array}\right) \div 2$$

per trading period, which amounts to ($5*(-20) + $20*(20)) = $300 per hour for a balanced FTR pattern (-20,20). So under Schedule 14.6, some rental will be paid into the FTR account.