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Estimating a New Zealand Electricity Emissions Factor[1]

Andy Philpott
Tony Downward

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for

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Executive Summary

In a recent study, Energy Modelling Consultants were engaged by the Ministry of the Environment to estimate the extent that electricity prices would be increased by a charge on CO_2 emissions by thermal plant. The model used in this study assumed a perfectly competitive market and risk neutrality, and estimated the increase in system short-run marginal cost (SRMC) of electricity that would arise from different levels of carbon charge. An annual average electricity emissions factor (EEF), defined to be the average increase in SRMC divided by the CO_2 charge, was calculated for each year in the period 2010-2032 under five scenarios corresponding to different choices of CO_2 emissions charges and dates for commissioning new generation plant. Energy Modelling Consultants estimated the EEF in 2010 for a CO_2 charge of \$20/tonne CO_2 to be 0.53 tonnes/MWh.

As discussed in the report by Energy Modelling Consultants, the SRMC values used to calculate EEF will underestimate wholesale prices in electricity markets with imperfect competition. To estimate markups from CO_2 charges in this setting, the EEF must be redefined as the increase in wholesale electricity price divided by CO_2 charge. This report describes the application of some simple Cournot equilibrium models to examine the effect that potential exercise of market power by large thermal plant in the New Zealand wholesale electricity market will have on changes in prices with a CO_2 charge. The models are calibrated to a selection of wet, dry and uncertain trading periods in 2008 (the most recent year in which full data are available in the Electricity Commission's Centralized Data Set), and then run under a number of different assumptions on the marginal cost increase faced by generators that are not large thermal generators.

We find:

(1) Under our assumptions on strategic bidding by generators, the EEF estimates depend on the relative frequencies of dry and wet hydrological conditions in New Zealand’s hydro lakes.

(2) Under our assumptions on strategic bidding by generators, estimates of EEF based on a selection of wet, dry and uncertain trading periods in 2008 differ from those computed under assumptions of perfect competition for 2010.

(3) Assuming that the carbon charge is \$12.50/tonne CO_2, our models estimate that the equilibrium average EEF under market conditions prevailing in 2008, can vary between 0.613 tonnes/MWh and 0.689 tonnes/MWh depending on the prevalence of wet hydrological conditions.

The results we present are based on a number of assumptions about the behaviour of market participants, and relative frequencies of types of trading period and hydrological conditions observed. Variations in these will give different results from those reported here. In particular we assume:

(1) Large thermal plant operate as Cournot players;
(2) Short-term electricity demand is inelastic and does not change in response to CO_2 charges;
(3) Hydro plant do not behave as Cournot players but adjust supply-function offers by uniformly increasing prices to reflect expected increases in opportunity cost;
(4) The offer behaviour of generators that are not treated as Cournot players is modelled using historical offer stacks from 2008.
(5) Residual demand defined by the demand minus the offers of generators that are not large thermal plants can be represented by curves with constant elasticity;
(6) Contract/retail positions are as estimated from observed generator behaviour in 2008 that is assumed to be in equilibrium, and these do not change in response to CO_2 charges.

1.0 Introduction

In a recent study [3], Energy Modelling Consultants estimated the extent that electricity prices would be increased by a charge on CO_2 emissions. The model used in this study assumed a perfectly competitive market and risk neutrality, and estimated the increase in the system short-run marginal cost (SRMC) of electricity that would arise from different levels of carbon charge.

The study described in [3] used the SDDP code [5] to study the long run evolution of SRMC as new generation plant and transmission capacity is commissioned and demand grows. SDDP constructs approximately optimal reservoir release policies that minimize the expected fuel and shortage cost over a planning horizon with uncertain reservoir inflows. These policies give the (approximately) optimal releases for a risk-neutral central planner who seeks to maximize social welfare.

The purpose of the study described in [3] was to estimate the increase in electricity prices that would result from charges on CO_2 emissions The statistic used to represent this is called the *electricity emissions factor* (EEF), defined to be the increase in SRMC divided by carbon cost. Formally,

$$\text{EEF} = \frac{\text{(SRMC with carbon charge - SRMC without carbon charge).}}{\text{carbon charge}}$$

As explained in [3], the system short-run marginal costs that emerge from such a study can be interpreted as electricity prices that might be observed in a perfectly competitive electricity market with risk-neutral agents. The New Zealand electricity market is not perfectly competitive, and agents are not risk neutral. When agents exercise market power, prices rise above SRMC. This means that EEF should be replaced by a *market emissions factor* EEF(m) that uses the wholesale electricity price (WEP) instead. Thus

$$\text{EEF(m)} = \frac{\text{(WEP with carbon charge - WEP without carbon charge).}}{\text{carbon charge}}$$

The purpose of this paper is to try and estimate an average EEF(m) for the year 2008[2] from historical observations of market behaviour. Our focus is thus to answer the question: "given the wholesale electricity market structure and transmission grid as in 2008, what would have been the expected increase in WEP from a charge on CO_2 emissions of $12.50/tonne?"

Since SRMC gives a lower bound on prices in a market with imperfect competition, it is tempting to suppose that one can prove *mathematically* that EEF(m) will always be higher than the value of EEF as defined in [3], and so that EEF is a *theoretical* lower bound on EEF(m). However, there are contrived examples (see [1]) where a charge on CO_2 emissions can be shown to increase SRMC and *decrease* WEP[3]. So the relative sizes of EEF(m) and EEF must therefore be settled by empirical modelling. This means that the setting and assumptions about agent behaviour must be formulated carefully to give realistic estimates.

In New Zealand, electricity prices fluctuate seasonally primarily due to the amount of hydro storage available. The plot in Figure 1 shows electricity prices at 6pm for three nodes (Haywards, Benmore and Otahuhu) over 2008. In this figure, we isolate three types of day during the year: on normal days prices typically remain under $100; on dry days, prices exceed $300; and in the lead in to a potentially dry period (uncertain days), prices are between $100 and $200.

[2] The year 2008 is the latest year for which the Electricity Commission Centralized Data Set [2] contains complete records of generator offers.

[3] These counterintuitive results are manifestations of "second best" welfare results which show that removing only one of several market imperfections can result in decreases in welfare. The examples involve a transmission line that is constrained in equilibrium, and a charge on CO_2 emissions results in a new equilibrium with no congestion and lower prices.

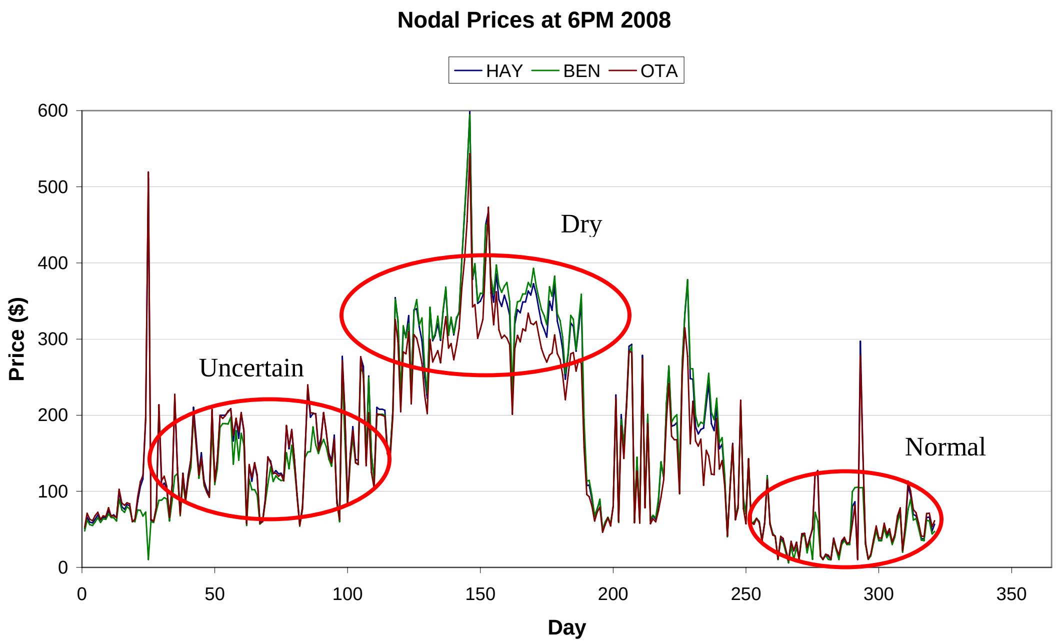

Figure 1: 6pm peak nodal prices for 2008.

In the analysis below, we select periods from each of these times of the year to determine how the effect of a carbon charge may differ depending on the type of day and type of trading period. We also consider "very wet" hydrological conditions in which prices are very low, except possibly in some high demand trading periods. An average EEF(m) for 2008 can then be estimated by weighting the average EEF(m) for each type of trading period by its frequency throughout the year.

In order to measure the relative frequency of these types of days over the longer term, we have extracted a price duration curve corresponding to offpeak (midnight) weekday periods over the time horizon January 1, 2005 - November 20, 2008 as shown in Figure 2.

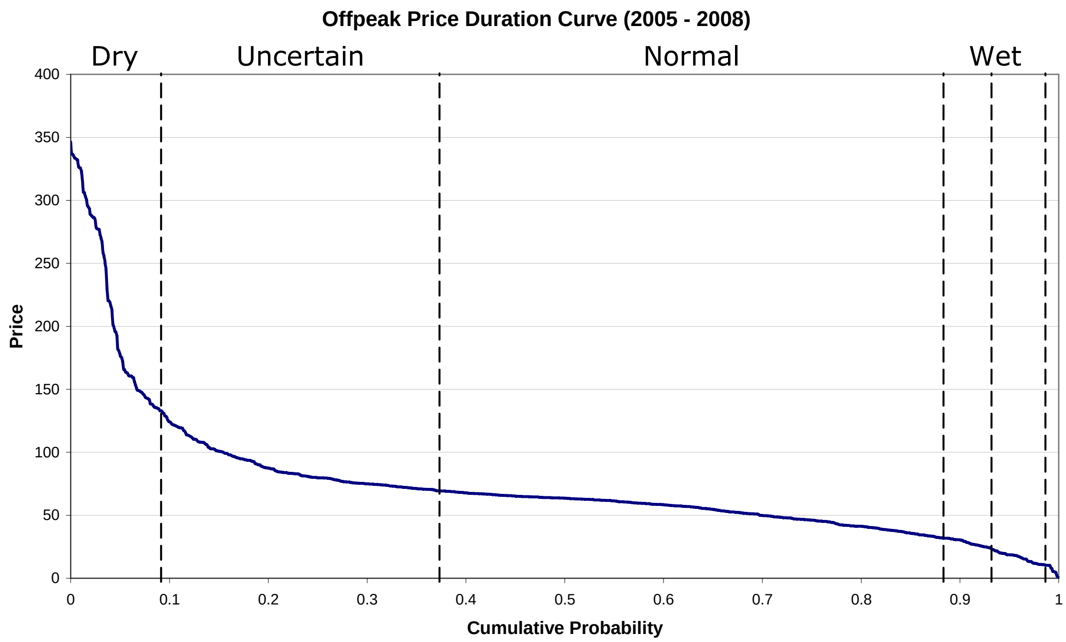

Figure 2: Offpeak Price Duration Curve 2005-2008.

We divide the offpeak prices into four groups indexed by i=wet, normal, uncertain, dry. This enables us to estimate the relative frequencies s(i) of the types of trading periods.

We define dry days as those with offpeak prices above $130. Uncertain days are ones with prices between $70 and $130, and we analyse three threshold prices for the normal/wet days: $30, $20 and $10[4].

The relative frequencies of each type of day are shown in Table 1 below for three different price thresholds for normal and wet years.

	s(i)		
	$30	$20	$10
Wet	0.110	0.060	0.011
Normal	0.510	0.560	0.609
Uncertain	0.284	0.284	0.284
Dry	0.096	0.096	0.096

Table 1: Relative Frequencies s(i) (estimated from 2005-2008).

Due to variations in demand, prices also fluctuate over the day. To model this intra-day variation we consider three demand levels

[4] Our purpose here is to enable a sensitivity test to be conducted on our 2008 results. Since 2008 was a dry year, we expect the EEF(m) for 2008 to correspond to a low frequency of wet days, and so a higher than average EEF(m).

indexed by p equal to peak, shoulder and offpeak. We select trading periods at 6pm to represent peak periods, midday to represent shoulder periods, and midnight to represent offpeak periods.

Using a similar method to that used to estimate values of s(i), we can compute the relative frequencies r(p) of peak, shoulder and off-peak periods.

For each trading period we compute an average price over 2008. This gives 48 numbers that are estimates of expected price in that period. By sorting these we arrive at a classification of periods into 16 offpeak periods between 10:30pm and 6:30am (trading periods 46-13), and 16 peak periods to be from 7:30am to 11:00am (trading periods 16-22) and 4:30pm to 9pm (trading periods 34-41) with the remaining periods classified as shoulder periods. This gives r(p) as shown in Table 2 below.

p	r(p)
Offpeak	0.333
Shoulder	0.333
Peak	0.333

Table 2: Relative Frequencies of Trading Periods.

2.0 **Our Model of NZEM**

The model we adopt assumes that the large thermal generators Contact Energy and Genesis Energy offer their thermal generation to the market as agents in a Cournot game. In general we assume that there is insignificant congestion in the transmission system[5]. The remaining generators that offer to the wholesale market are not modelled as strategic Cournot players, but are considered to behave in aggregate as they did before the CO_2 charges were imposed, however with an increase in price of the residual demand stack. This means that they are strategic inasmuch as their original observed offers represent strategic behaviour, but they do not alter this in response to changes in offers of the large thermal plants, apart from increasing the price of their offers by a fixed markup, K.

Although they comprise a mix of generation technologies, all the generators that offer to the wholesale market apart from the large thermal plants operated by Contact and Genesis are referred to as "hydro plants", since they include the generating stations of Meridian Energy, Mighty River Power, the Clutha system, Waikaremoana, and the Tongariro Power Development as well as all the plant operated by Trustpower, which includes several hydro schemes[6].

2.1 Data

The data that we use in our models are taken from the report [3] and are as follows.

Fuel Costs and Carbon Content

Fuel Type	Price ($ / GJ)	Tonnes CO_2 / GJ
Gas	6.0	0.0528
Coal	4.0	0.0912

Table 3: Fuel properties (source [3]).

[5] Since transmission congestion complicates the calculation of EEF(m) considerably, we have chosen not to include this feature. Preliminary experiments in some sample dry periods show that congestion gives EEF(m) values that vary with location.

[6] Note that the offering stations in the Centralized Data Set apart from the large thermal plants operated by Contact and Genesis includes geothermal stations, wind farms, and smaller thermal plant like Whirinaki and Southdown, so these are also included in the description "hydro plants".

Plant Properties

Plant	Fuel Type	Heat Rate (GJ/MW)	Capacity (MW)
Huntly	Coal	10.50	972
E3P	Gas	7.08	385
P50	Gas	9.50	50
Taranaki CC	Gas	7.30	377
Otahuhu B	Gas	7.05	404

Table 4: Plant properties (source [3]).

Offer Stacks of Hydro Plants

For the trading periods we study, the offer stacks of all hydro generators (i.e. those plant not listed in Table 4) are extracted from the Electricity Commission Centralized Data Set [2] and aggregated, to yield a combined offer stack as shown in Figure 3.

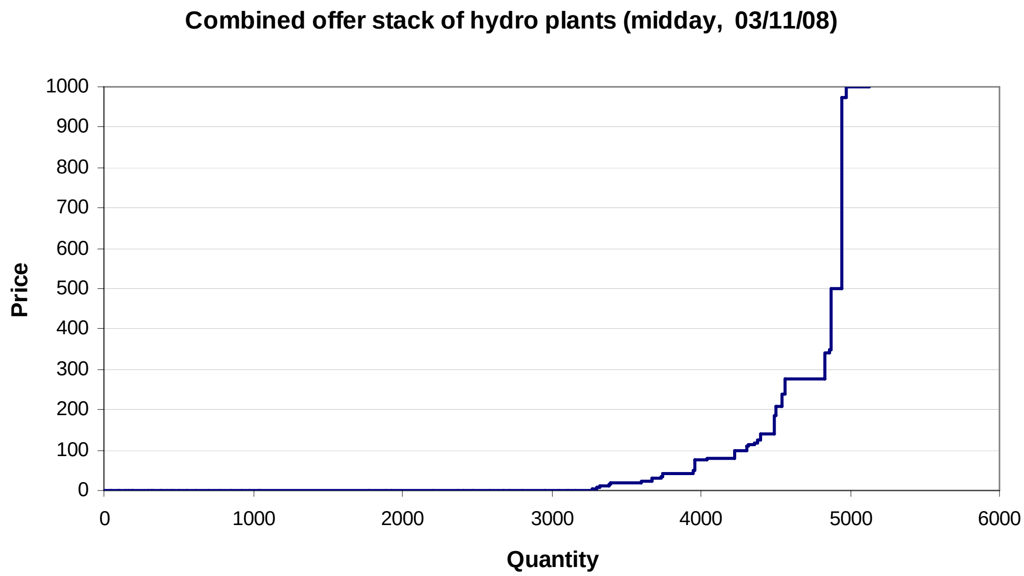

Figure 3: Hydro plants' aggregated offer stack.

Demand

The demand for each trading period has also been extracted from [2]. We treat the wholesale market as a single node and assume that total demand is inelastic in the short term. To account for transmission losses we estimate an effective demand from the offer stacks and the actual prices; this inflates the national demand by between 5% and

13% depending on the time of day or year. The inflation factor is chosen to reproduce the observed prices from the offer stacks.

2.2 Calibration to 2008 observed prices

Before one can estimate the value of EEF(m), the Cournot model we use must be calibrated to verify that it predicts observed prices without a CO_2 charge.

To do this the total observed demand and the aggregated offer curve of plants not in Table 4 are combined to form a stepped *residual demand curve* that is then approximated by a smooth curve. In most trading periods the residual demand curve is approximated well by a curve with constant elasticity as shown in Figure 4.

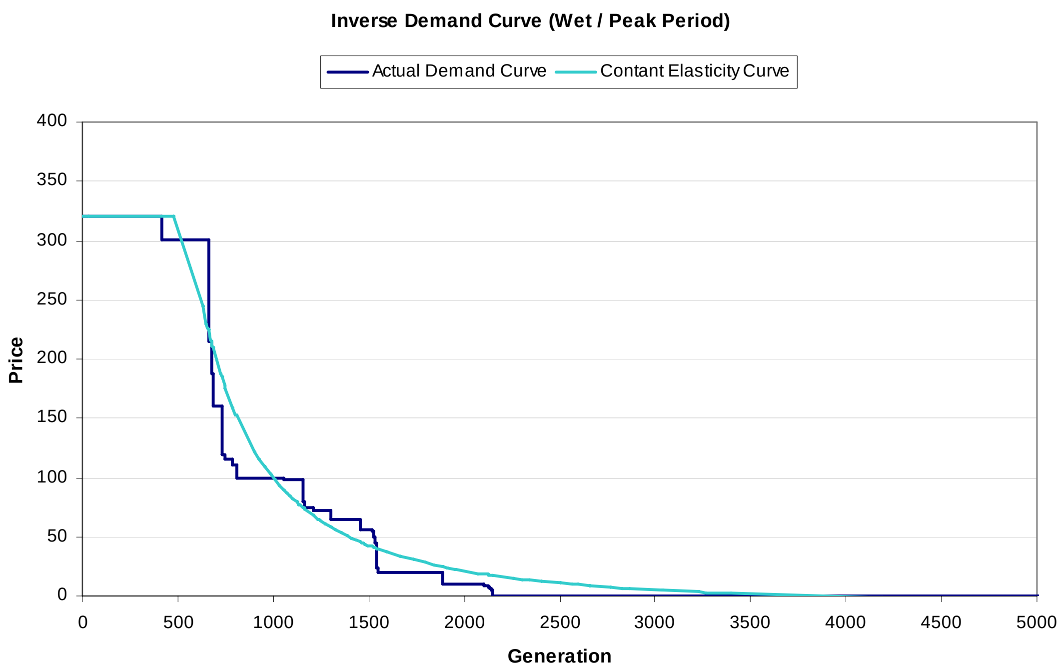

Figure 4: Constant elasticity demand curve.

Thus residual demand D(p) at price p is modelled as

$$D(p) = D_0(\frac{p}{p_0})^{-\gamma}$$

where $\gamma>0$ is the price-elasticity of residual demand.

For a given trading period we then estimate the level of retail load and contracting that Contact and Genesis are facing in the trading period, and construct a Nash-Cournot equilibrium for this period[7]. The estimation of contract levels in 2008 is based on published information about each company's load obligations [10] and observations on the offer stack of each generator. It is well-known (see e.g. [4]) that in a supply-function equilibrium, a generator with a contract for differences of quantity Q will offer generation at a price below marginal cost up to Q, and then bid with a positive markup above Q. An example of this behaviour is illustrated for a generator with constant marginal cost in Figure 5.

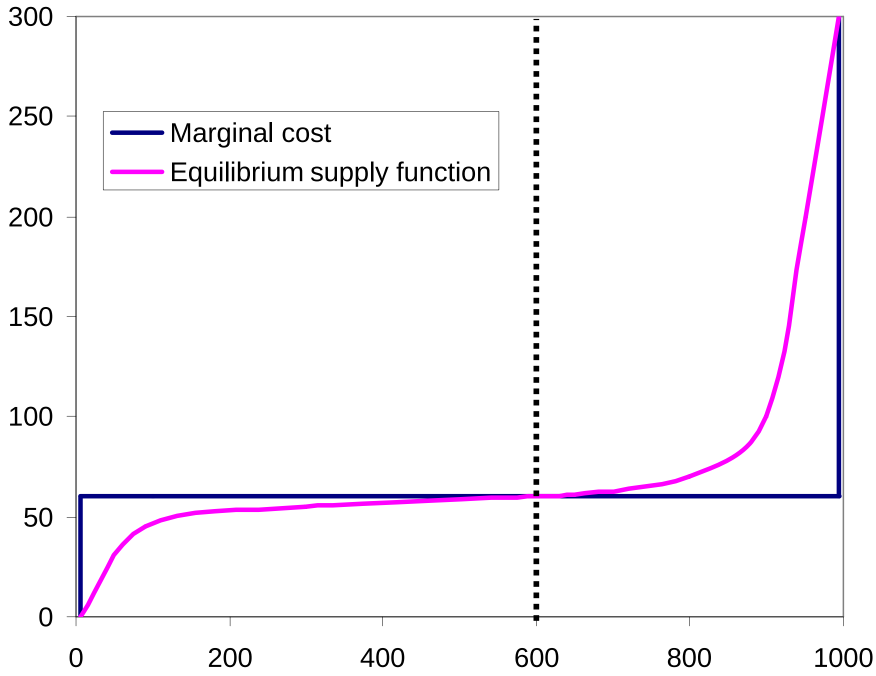

Figure 5: Example shape of an equilibrium supply function. Here capacity is 1000MW, Q = 600 and marginal cost = $60.

This enables us to use the marginal cost and bidding behaviour seen in historical offer stacks to estimate the contract and retail exposure of each generator in the trading period we are studying.

The results of this calibration exercise are the following estimated contract/retail levels.

[7] The procedure for doing this is described in Section 2.3. We assume in our analysis that contract/retail quantities are not changed by the addition of a CO_2 charge.

	Genesis	Contact
Off Peak	500 MW	150 MW
Shoulder	750 MW	300 MW
Peak	850 MW	400 MW

Table 5: Estimated contract/retail levels in normal periods.

	Genesis	Contact
Off Peak	500 MW	250 MW
Shoulder	750 MW	400 MW
Peak	900 MW	550 MW

Table 6: Estimated contract/retail levels in uncertain periods.

	Genesis	Contact
Off Peak	500 MW	400 MW
Shoulder	750 MW	500 MW
Peak	850 MW	600 MW

Table 7: Estimated contract/retail levels in dry periods.

The Nash-Cournot equilibrium with the estimated contract level is computed for each trading period. The price predicted from this is then compared with the observed price and used to make small adjustments to the contract level and residual demand curve in order to give a close match in prices.

2.3 Solving Nash-Cournot Equilibrium Problems

In our experiments Nash-Cournot equilibrium problems are solved on normal, uncertain and dry days, each for peak, shoulder and offpeak periods. Since results vary, we sample N=25 weekdays from each type of day, and average the EEF(m) values over these 25 outcomes.

The dry periods were taken from five consecutive weeks in May-June 2008, the normal periods were taken from five consecutive weeks in October-November 2008 and the uncertain periods were taken from five consecutive weeks in March-April 2008.

This gives a total of 225 equilibrium problems. For each problem the price is given by a residual demand curve (discussed in section 2.2). Genesis and Contact then choose their thermal plants' generation levels so as to maximize their respective profits. The equilibrium is computed using a *fictitious play* approach (see e.g. [9]), whereby each firm optimizes assuming the other is fixed. This is repeated until the generation levels converge to a fixed point – the Nash equilibrium.

For purposes of illustration, suppose that there is no markup in prices from the hydro plants in response to a carbon charge (K=0). If a carbon charge is added this will increase the thermal plants' fuel costs yielding to a new equilibrium point with a different price. The EEF(m) for each period can then be computed from the formula on page 5.

Figures 6 – 14 plot computed EEF(m) against the historically observed price, for a carbon charge of \$12.50 / tonne CO_2, for each of the 225 periods sampled. Each plot shows N= 25 points.

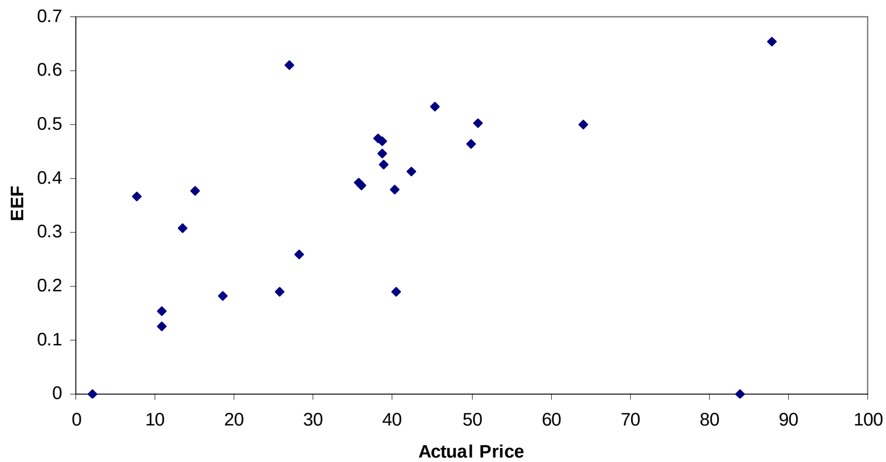

Figure 6: EEF(m) vs Price – Normal, offpeak periods, K = 0.

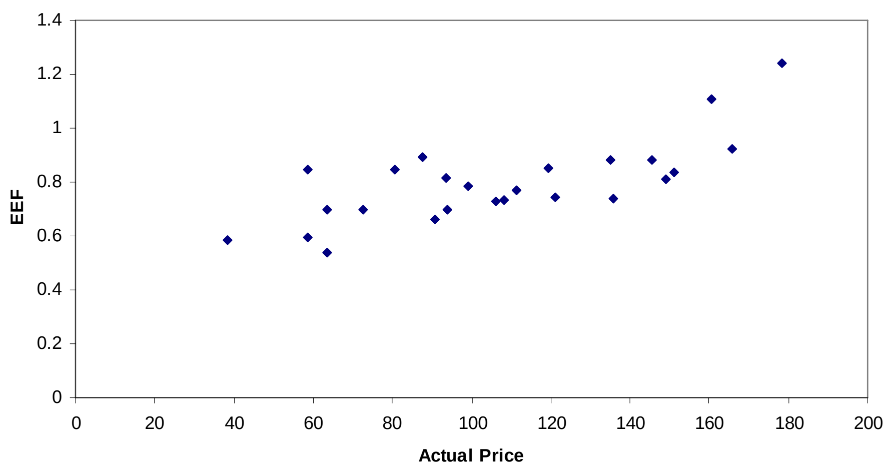

Figure 7: EEF(m) vs Price – Uncertain, offpeak periods, K = 0.

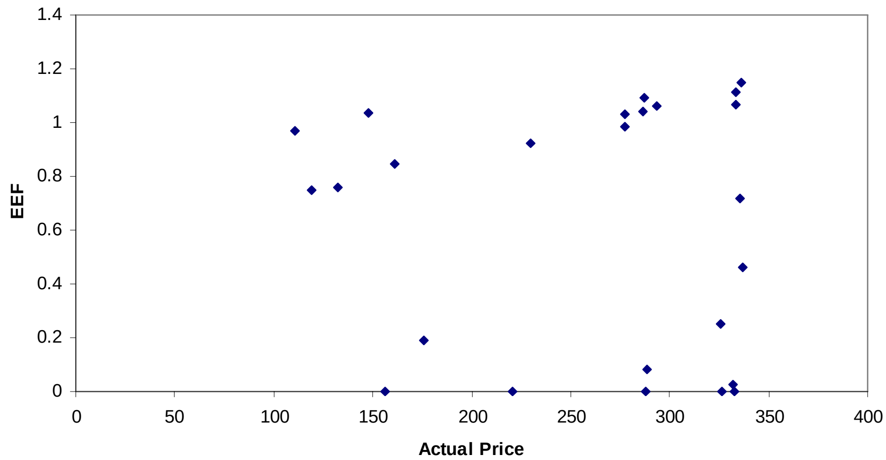

Figure 8: EEF(m) vs Price – Dry, offpeak periods, K = 0.

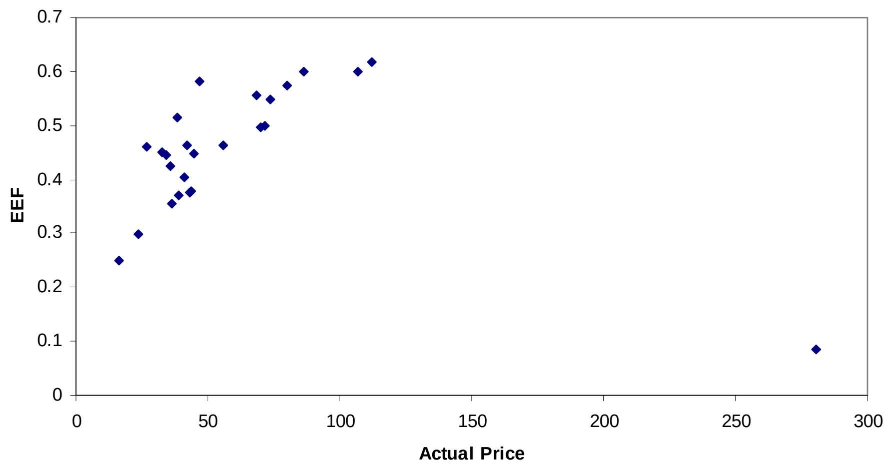

Figure 9: EEF(m) vs Price – Normal, shoulder periods, K = 0.

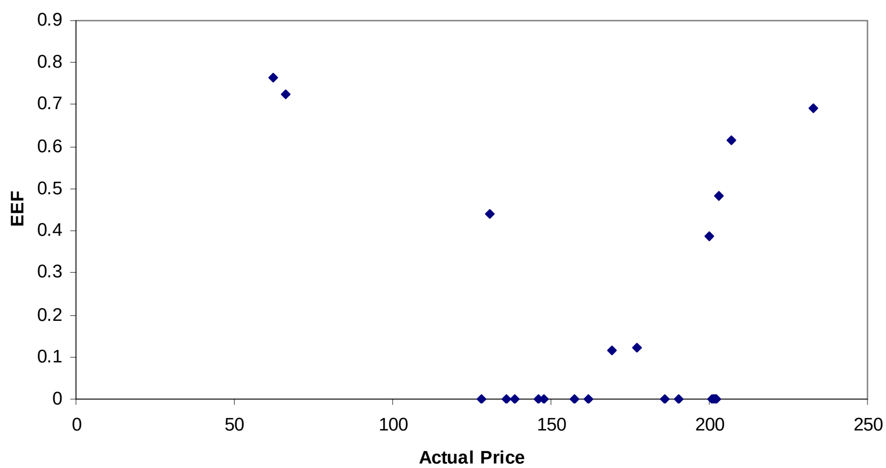

Figure 10: EEF(m) vs Price – Uncertain, shoulder periods, K = 0.

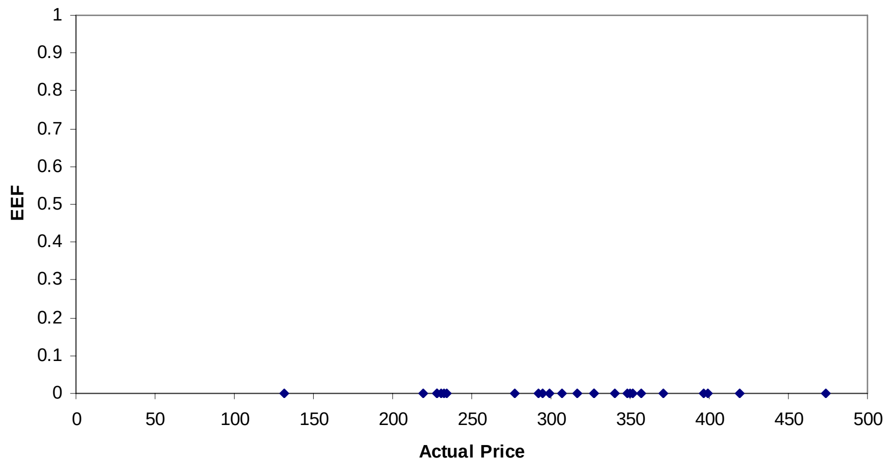

Figure 11: EEF(m) vs Price – Dry, shoulder periods, K = 0.

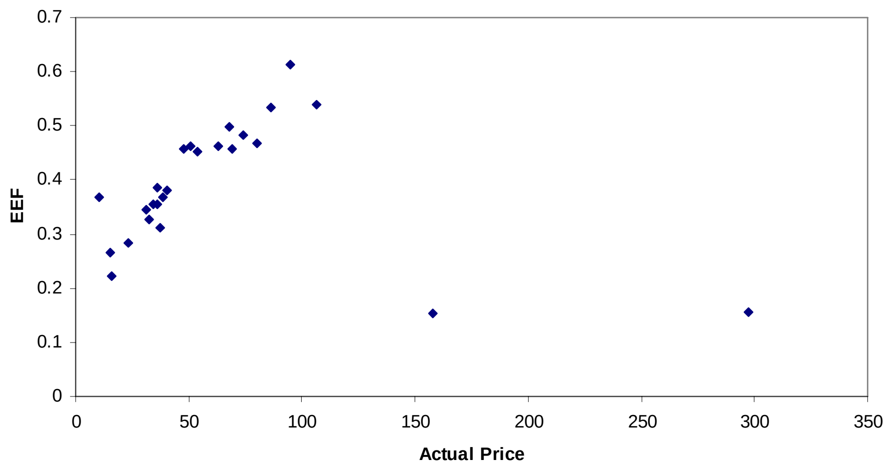

Figure 12: EEF(m) vs Price – Normal, peak periods, K = 0.

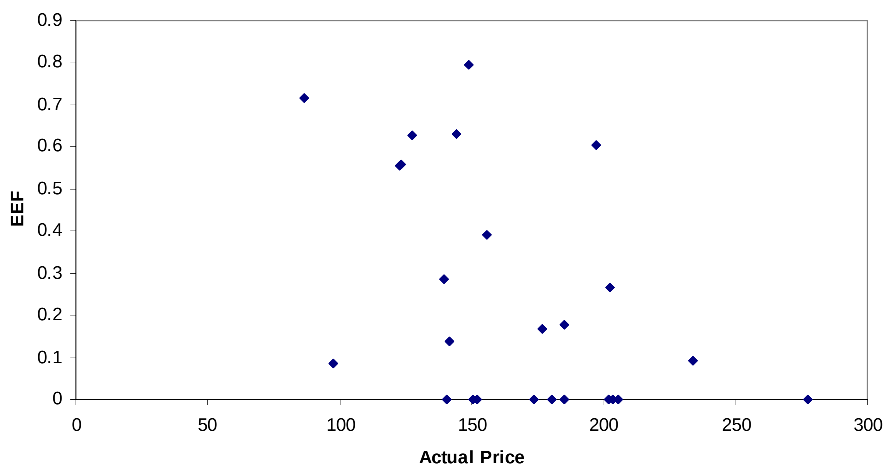

Figure 13: EEF(m) vs Price – Uncertain, peak periods, K = 0.

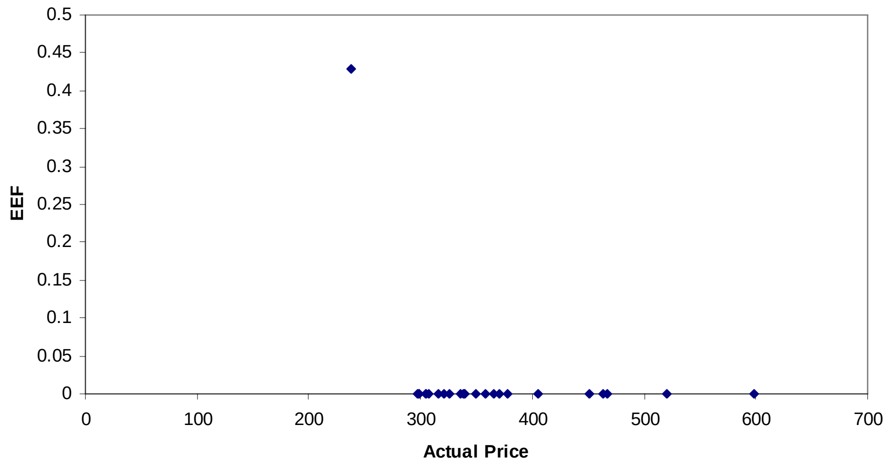

Figure 14: EEF(m) vs Price – Dry, peak periods, K = 0.

2.4 Estimation of K

In this section we discuss the choice of K we use in our model. Recall that K is the markup that we expect to see in the offer stack corresponding to the generators that are not Cournot players, when CO_2 charges are imposed on thermal plants.

The generators in our model that are not Cournot players are mainly hydro generators[8]. If we ignore river-valley constraints, cascading reservoirs, resource consents etc. then the marginal cost function for a hydro generator contains a single tranche with quantity equal to the capacity of the hydro plant, and price equal to its marginal water value π. In the absence of constraints a price-taking hydro operator would offer this tranche to the market, so as to generate at zero if prices are below π, and generate at its capacity if prices exceed π.

In practice the stack that each hydro generator offers is not perfectly competitive. It can be shown in the theory of supply-function equilibrium (see e.g. [4]) that generators offer at less than marginal cost up to their contract point Q, and mark up their offer above marginal cost beyond Q. A stylized example of such a curve is shown in Figure 5, reproduced as Figure 15 below.

[8] Recall that they consist of all offering generators listed in the CDS apart from those in Table 4, so they include the plant of Meridian Energy, Mighty River Power, the Clutha system, Waikaremoana and the Tongariro Power Development as well as all the plant operated by Trustpower, which includes several hydro schemes.

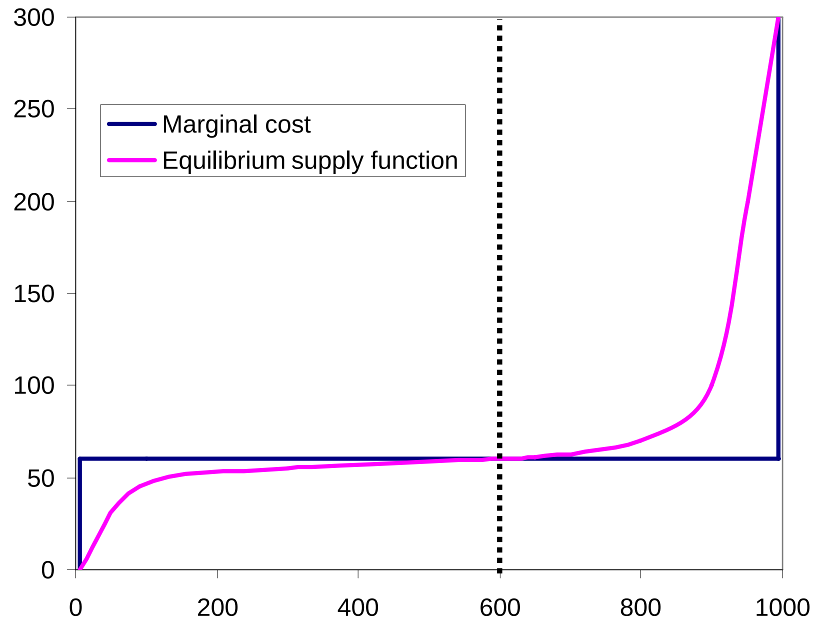

Figure 15: Example shape of an equilibrium supply function. Here capacity is 1000MW, contract quantity = 600 and marginal cost = $60

Consider such an offer curve for a hydro generator. If its marginal cost increases, then according to the theory, the equilibrium supply function for the generator will increase at the contract point Q by the increase in marginal cost faced by the generator. In Figure 6 this means that the step curve for the generator shifts up by the increase in marginal cost and the equilibrium supply function adjusts accordingly.

We assume therefore that each hydro generator's supply function moves up by the increase K in marginal water value that arises from a CO_2 charge. Assuming equilibrium, this increase of K is exact on the tranche corresponding to the contract position of the generator.

We make the assumption that the increase is equal to K at all points on the offer stack of each hydro generator, and therefore equal to K at all points on the aggregated stack of all the hydro generators. This is illustrated in Figure 16.

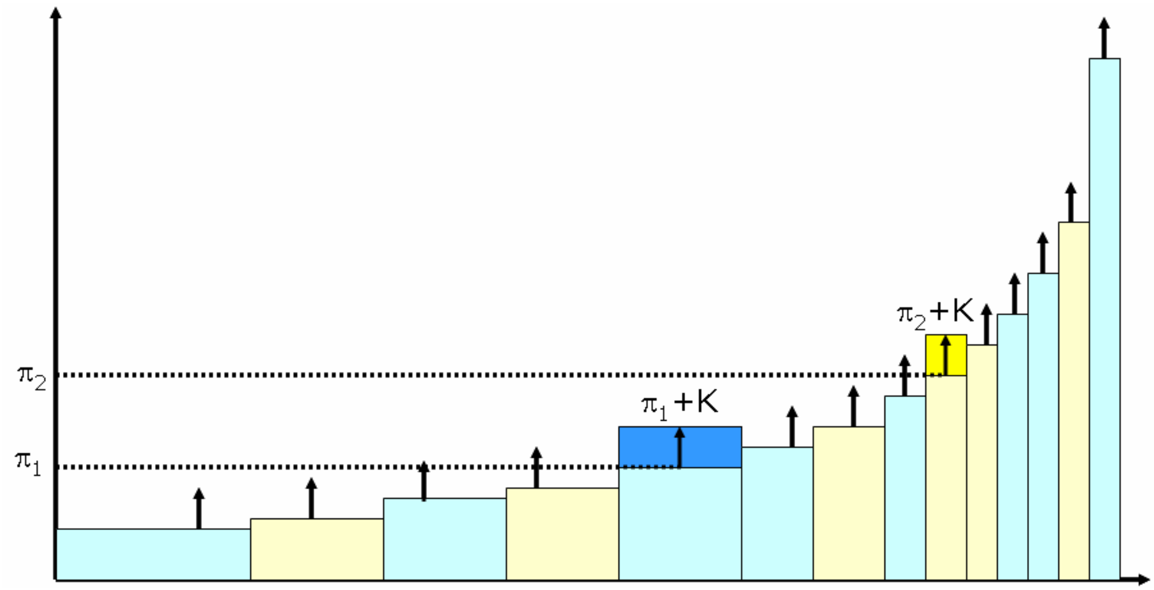

Figure 16: Example of two offer stacks aggregated. In equilibrium offer prices at marginal costs π_1 and π_2 would move up by K. We assume all other offer prices increase by K also.

It remains to estimate the magnitude of the markup K that arises from a CO_2 charge. The marginal water value in a market with imperfect competition is different from that in a perfectly competitive market. Both values represent an opportunity cost of using water now rather than delaying its release. In the market this opportunity cost is defined in terms of future electricity prices (rather than displaced thermal/shortage costs). This makes the estimation of opportunity cost a challenging problem[9].

Our approach is to model the markup in water value K in terms of the expected increase in future prices for electricity i.e. the future EEF(m) values). The expected increase in future prices of electricity results in an increase K in marginal water value that depends on the hydrological conditions that are being experienced at the time. Thus we define a different markup K_i for each hydrological state i = wet, normal, uncertain, or dry, and define

E(K,i,p) = average EEF(m) in state i and period type p when hydro markup K_i equals K.

[9] Game-theory models for constructing these estimates using a form of backwards induction on reservoir state variables have been developed and implemented in code (see e.g. Scott and Read [8]) but the existence and uniqueness of Nash equilibria in such multi-stage problems is not guaranteed in general.

where i = wet, normal, uncertain, dry.

The complication that this raises is that the markup in each state not only arises from the expected future EEF(m), but it is also used in the computation of EEF(m) when solving for the Nash-Cournot equilibrium. This circularity results in a fixed-point computation that we now describe in more detail.

An extra unit of water available in hydrological state i may be used in a future hydrological state j and trading period p, where p = offpeak, shoulder and peak. We define

P(i,j,p) = proportion of future trading periods of type p and hydrological state j that an extra unit of water will be used in to generate power given that the system is in hydrological state i[10].

The proportions P(i,j,p) arise from hydro generators optimizing the use of stored water, and so estimating these quantities accurately needs an optimization model for hydro-generators (that will also involve expectations of future prices in the wholesale market).

For simplicity, we assume that P(i,j,p) has the following form:

P(i,j,p)	= r(p),	if i = wet and j = wet,
	= r(p)s(j),	if i = normal or uncertain,
	= r(p),	if i = dry and j = dry,
	= 0,	otherwise.

Recall that r(p) gives the proportion of trading periods of type p, and s(j) gives the probability of observing system hydrological state j. Our assumption about P(i,j,p) means that:

1. If the system is in a wet state then any additional water is certain to be used in that state. In other words additional water has probability zero of being used in a normal, uncertain or dry state. Since in a wet state, additional water is almost certain to be used immediately or spilt as the reservoirs are close to full, this is a reasonable approximation.

[10] The parameters P(i,j,p) are NOT transition probabilities of a Markov chain, but represent steady-state probabilities of the use of water based on an optimal decision process.

2. If the system is in a dry state then any additional water is certain to be used in that state. In other words additional water has probability zero of being used in a normal, uncertain or wet state. Since in a dry state, additional water is almost certain to be used immediately in that state or kept for an even drier state, this is a reasonable approximation.

3. If the system is in a normal or uncertain state then any additional water will be used in one of the four hydrological states depending on their relative likelihood[11].

Based on this approximation we can compute the markup K for each hydrological state by solving the following system:

$$K_{wet} = 0$$
$$K_{dry} = \Sigma_p P(dry,dry,p)E(K_{dry},dry,p)$$
$$K_{normal} = \Sigma_j \Sigma_p P(normal,j,p)E(K_j,j,p)$$
$$K_{uncertain} = \Sigma_j \Sigma_p P(uncertain,j,p)E(K_j,j,p)$$

Observe that since P(normal,j,p) = P(uncertain,j,p), the solution to the system will satisfy $K_{normal} = K_{uncertain}$. This means that the solution can be computed as a sequence of one-dimensional fixed-point iterations.

If the day is wet then we assume that it is difficult to retain water for later use. Thus we set $K_{wet} = 0$ for this case. Any nonzero values of EEF(m) in the wet state come about from thermal capacity being required in some peak periods where it corresponds to the increase in gas cost from a CO_2 charge. For these periods, we assume that the peaking thermal plant will be a gas turbine[12]. For a CO_2 charge of $12.50 this gives an EEF(m) of approximately 0.385 tonnes/MWh in a wet/peak period.

Since the computation of E(K,j,p) requires in each case the numerical solution of N=25 Nash-Cournot equilibrium problems, we cannot solve the fixed-point iteration analytically for dry, uncertain or normal markup values. The calculation of K_{dry}, K_{normal}, and $K_{uncertain}$ thus requires an iterative procedure as follows:

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[11] We assume here that being in an uncertain state does not make dry future periods more likely than being in a normal state, although this could be an extension of our model.

[12] We assume that the gas plant is on the margin in these periods and all plant are bidding competitively, so EEF(m) will correspond to an increase in SRMC in this case.

Computation of K_{dry}

Step 0: Set K=0.

Step 1: Increase prices in the residual demand curve by K and solve for Nash-Cournot equilibrium in each of the N sample cases in dry peak, dry offpeak and dry shoulder periods.

Step 2: Compute the average EEF(m) for each of dry peak, dry offpeak and dry shoulder periods.

Step 3: Set K= weighted average of the EEF(m) values from Step 2.

Step 4: If K has not converged then go to Step 1.

The result gives a value of K for dry days that has the property that if it is added to the residual demand curve for dry peak, shoulder, and offpeak periods, and if EEF(m) is computed for each of these periods, then the weighted average of EEF(m) equals the markup K.

Computation of $K_{uncertain}=K_{normal}$

Step 0: Set K=0.

Step 1: Increase prices in the residual demand curve by K and solve for Nash-Cournot equilibrium in each of the N sample cases in normal peak, normal offpeak and normal shoulder periods, and for each of the N sample cases in uncertain peak, uncertain offpeak and uncertain shoulder periods.

Step 2: Compute average EEF(m) for normal peak, normal offpeak and normal shoulder periods, and for uncertain peak, uncertain offpeak and uncertain shoulder periods.

Step 3: Set K = weighted average of the EEF(m) values from Step 2 and the EEF(m) values already computed for dry and wet days.

Step 4: If K has not converged then go to Step 1.

The result gives a value of K for normal or uncertain days that has the property that if it is added to the residual demand curve for these sorts of days, then the average EEF(m) computed over all types of periods returns the markup K. Observe that this final value of K is the same as the average EEF(m) for the CO_2 charge being considered, and is thus the value that we report for this case.

3.0 **Results**

As shown in Table 1, we have allowed for different proportions of wet periods by changing the price threshold. We will first present results corresponding to values of s(i) corresponding to a normal/wet price threshold of $20. We assume throughout this section that the CO_2 charge is $12.50/tonne. This gives an average EEF(m) of 0.65 tonnes/MWh as follows.

Wet Periods

The value of EEF(m) for wet periods is zero except in peak periods where it corresponds to the increase in gas cost from a CO_2 charge.

For these periods, we assume that the peaking thermal plant will be a gas turbine. The EEF(m) for wet/peak periods is therefore found from the heat rate of a gas turbine (e.g. Taranaki CC) multiplied by the carbon content of gas:

$$E(K_{wet},wet,peak) = 7.3 \text{ GJ/MWh} \times 0.0528 \text{ tonnes/GJ} = 0.385 \text{ tonnes/MWh.}$$

Dry Periods

For dry periods K_{dry} converges to 0.854 tonnes/MWh[13].

Normal/Uncertain Periods

Using the estimates of s(i) and r(p) from Tables 1 and 2, K_{normal} converges to 0.65 tonnes/MWh. This is the same value as $K_{uncertain}$. Given these values of K we can compute EEF(m) at convergence for each type of day and period. This gives the values shown in Table 8.

[13] This figure is based on the assumption that if the system is in a dry state then any additional water is certain to be used in that state. Thermal plants are generally at capacity in this hydrological state except in offpeak periods where they withhold more as CO_2 charges are applied. The resulting increase in average price in these periods means that hydro generators mark up their supply curves, leading to new Nash-Cournot equilibria in each sample period. This mechanism is quite sensitive to the available capacity of thermal plant. If all thermal plant are at capacity in all offpeak periods then for any K, E(K,dry,p) is determined by the hydro stack for all periods p and so equals K, meaning that there are infinitely many solutions to our recursion for K, resulting in an infinite range of EEF(m) values. However even if all thermal plant are at capacity in offpeak periods, and we were to choose $K_{dry}=0$ as the lowest possible equilibrium markup, then we obtain a overall estimate of EEF(m) of 0.56 tonnes/MWh.

	Off Peak	Shoulder	Peak
Wet	0.000	0.000	0.385
Normal	0.638	0.665	0.667
Uncertain	0.680	0.659	0.695
Dry	0.854	0.854	0.854

Table 8: EEF(m) values for different types of day and period.

Based on the estimates of s(i) and r(p) from Tables 1 and 2, the overall average EEF(m) for this experiment is 0.65 tonnes/MWh (i.e. the same value of K for normal/uncertain periods).

For the equilibrium values of K, the distributions of EEF(m) against price for each type of day and period are shown in Figures 17 – 25.

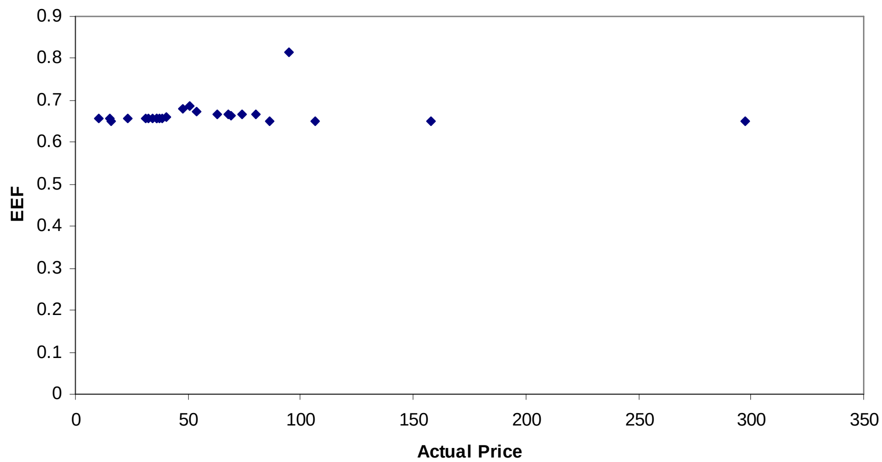

Figure 17: EEF(m) vs Price – Normal, peak periods.

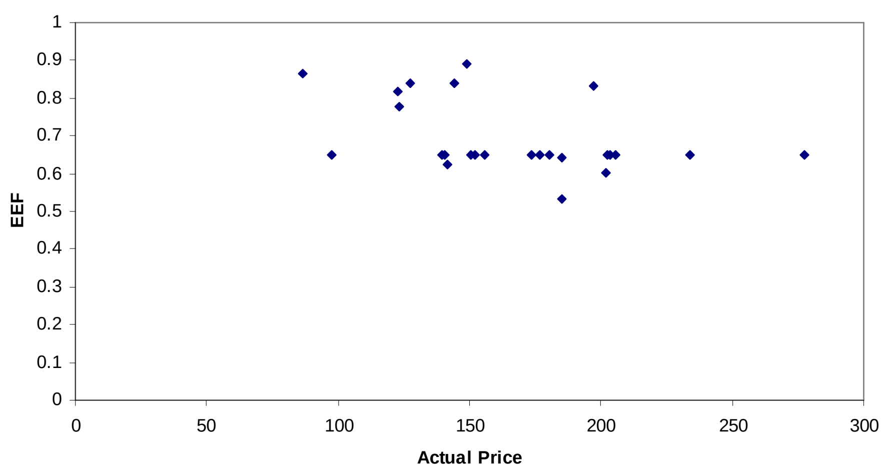

Figure 18: EEF(m) vs Price – Uncertain, peak periods.

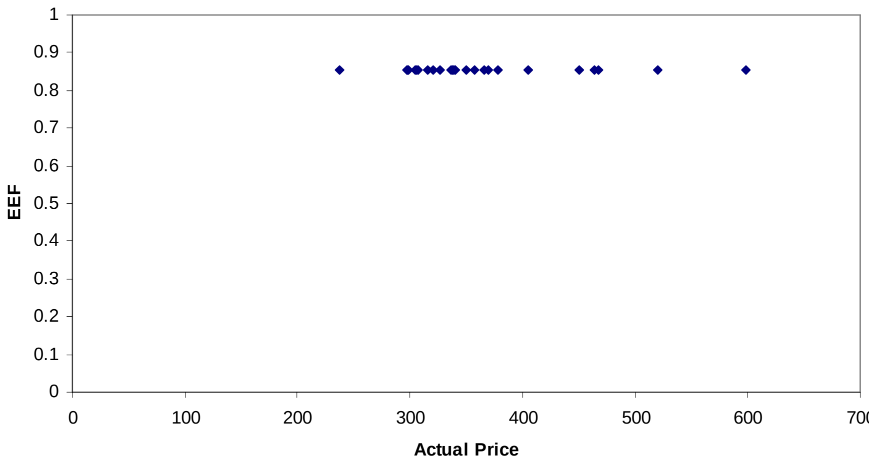

Figure 19: EEF(m) vs Price – Dry, peak periods.

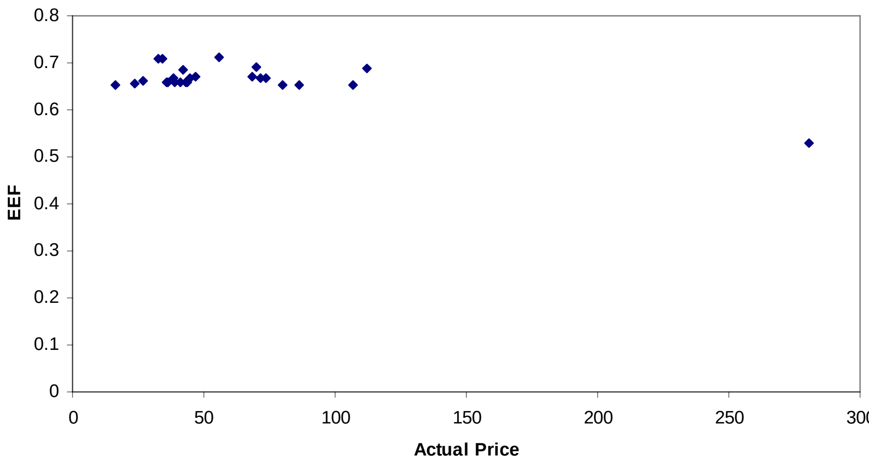

Figure 20: EEF(m) vs Price – Normal, shoulder periods.

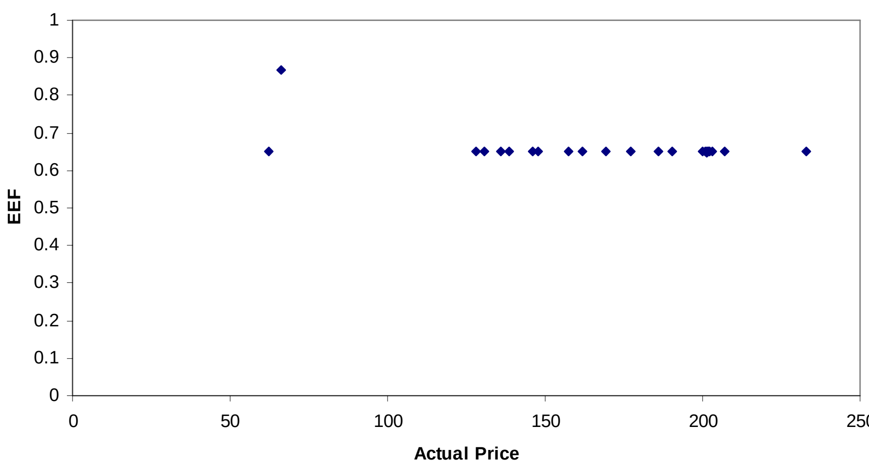

Figure 21: EEF(m) vs Price – Uncertain, shoulder periods.

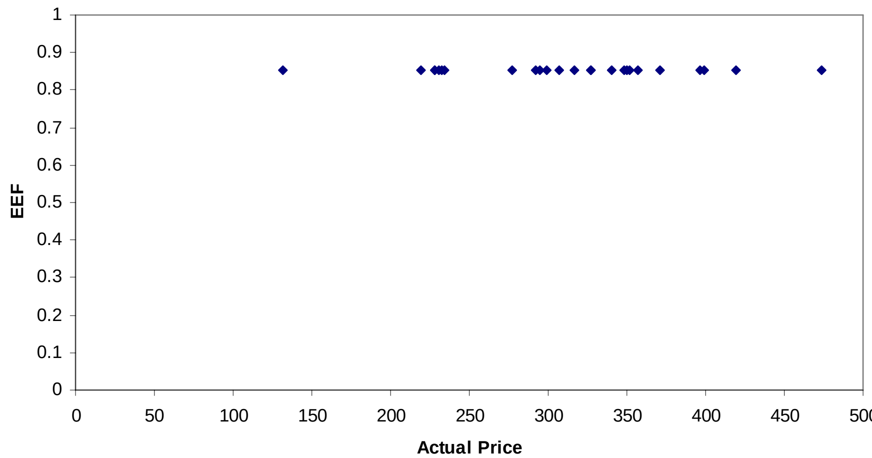

Figure 22: EEF(m) vs Price – Dry, shoulder periods.

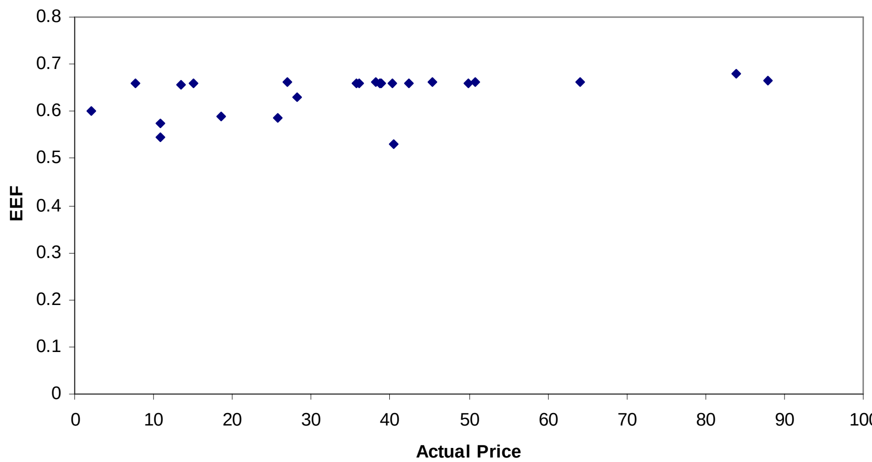

Figure 23: EEF(m) vs Price – Normal, offpeak periods.

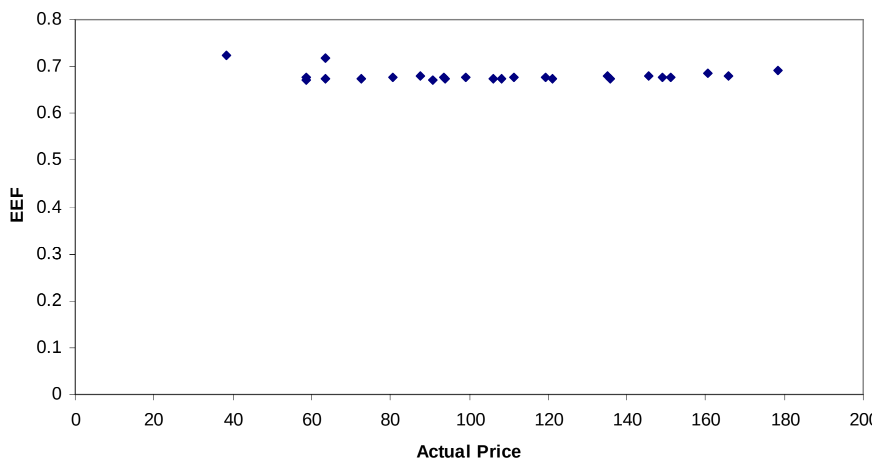

Figure 24: EEF(m) vs Price – Uncertain, offpeak periods.

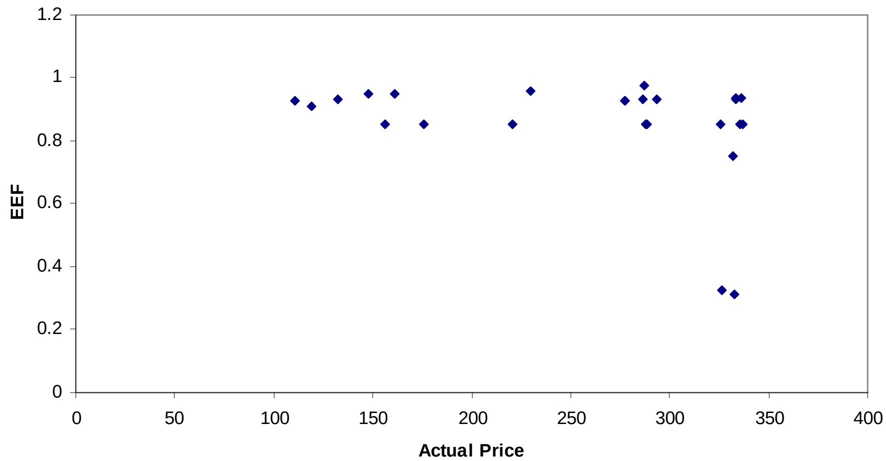

Figure 25: EEF(m) vs Price – Dry, offpeak periods.

Observe that the final value of average EEF(m) depends on the relative frequencies of wet and dry hydrological conditions. If there are fewer wet days then the average EEF(m) value will increase. To test this computationally, we recomputed EEF(m) with different values of s(i) taken from Table 1.[14]

First, if we assume that the threshold price between normal and wet is $30, then this increases the proportion of wet days (from 0.060 to 0.110). This leads to a lower EEF(m) of 0.613 tonnes/MWh.

Now, if we assume that the threshold price between normal and wet is $10, then this reduces the proportion of wet days (from 0.060 to 0.011). This leads to a higher EEF(m) of 0.689 tonnes/MWh.

4.0 Discussion and Extensions

This report presents a new perspective on the impact of CO_2 charges on New Zealand wholesale electricity prices. Experiments with a model of strategic behaviour by generators applied to a selection of wet, dry and uncertain trading periods in 2008, predict higher markups under CO_2 charges than those predicted by Energy Modelling Consultants for 2010 under a central planning model.

[14] Observe that it is not possible in these experiments to simply weight the EEF(m) values in Table 8 with new probabilities, since K_{normal} and $K_{uncertain}$ will be different, leading to different EEF(m) values in Table 8. A completely new set of Nash-Cournot equilibria must be computed.

The results in our report are based on some key modelling assumptions. For computational convenience we assume a Cournot duopoly consisting of large strategic thermal generators. The Cournot model means that each thermal generator injects a fixed quantity to the market assuming that its competitor also does the same (and does not vary its input with price as it would with a supply-function offer).

We treat the other generators in aggregate as offering a fixed supply function assumed to be the aggregate of optimal hydro offers. When opportunity costs increase by K, we assume that the optimal supply-function offers move up uniformly by K (at least in aggregate). We do not assume that the shape of this curve alters strategically to respond to the Cournot players' offers.

We have not attempted to give estimates of the statistical reliability of our results – these are difficult to obtain when conjectures are being made about the behaviour of electricity market participants and the degree to which they exploit market power opportunities. We observe however that our results can be quite sensitive to assumptions on the relative frequencies of different hydrological conditions and whether thermal plant are at capacity in equilibrium in dry hydrological conditions.

There are obvious extensions to this work that attempt to include transmission constraints. These complicate the analysis in several ways. Contract/retail positions become dependent on location, and the agents have a set of location-dependent actions in equilibrium. Moreover with possible transmission congestion, the equilibria calculation for agents under the assumption of perfect rationality becomes more difficult, to the extent that pure-strategy equilibria may fail to exist or be unique. From a policy perspective, we have found in cases where equilibria are computable that transmission constraints can lead to different estimates of EEF(m) for different locations.

Of perhaps more importance than including transmission constraints is a detailed estimation of the probabilities P(K,i,j,p). Marginal water values that give opportunity costs of generation come from hydro-generators' reservoir optimization policies. We have not modelled these in this report and for simplicity assumed that the mixture of outcomes from the Cournot equilibria under different hydrological conditions are essentially independent of these policies. A more comprehensive analysis would need to account for them more thoroughly.

Finally, our model uses the historical offer stacks from 2008 to construct residual demand curves for our Cournot model. To estimate EEF(m) in future years, one might argue that these offer stacks reflect a range of possible hydrological conditions, and so they can be used in such a model with an appropriate change in s(i). Nevertheless the accuracy of predictions of EEF(m) for future years will depend on the extent that these historical offers represent generator behaviour in the future.

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